Splot liniowy

Jeżeli

$$x[n] \Leftrightarrow X(z) \text{ z ROC } |z| > R_x \text{ i } h[n] \Leftrightarrow H(z) \text{ z ROC } |z| > R_h$$

to

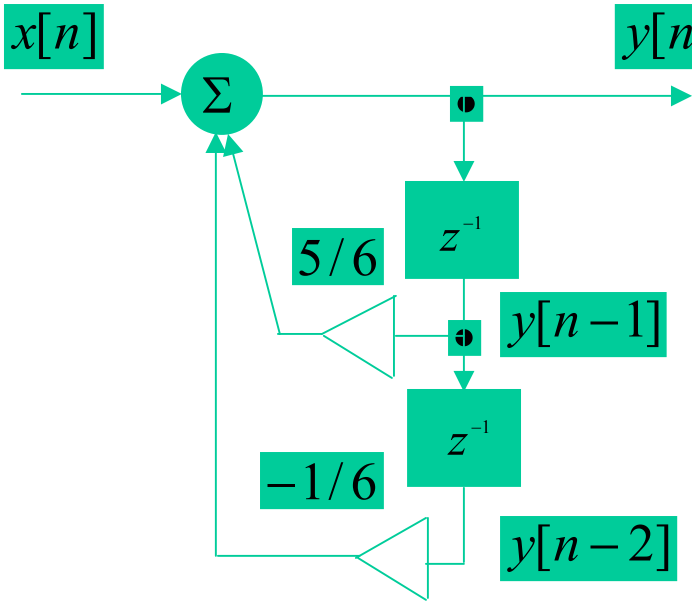
$$x[n] * h[n] \Leftrightarrow X(z)H(z) \text{ z ROC } |z| > \max(R_x,\ R_h)$$

gdzie

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] \quad \Leftrightarrow \quad X(z) \longrightarrow \boxed{H(z)} \longrightarrow Y(z)$$

Przykład



$$y[n] = x[n] + \frac{5}{6} y[n-1] - \frac{1}{6} y[n-2]$$

$$Y(z) = X(z) + \frac{5}{6} z^{-1} Y(z) - \frac{1}{6} z^{-2} Y(z)$$

$$H(z) \stackrel{\Delta}{=} \left.\frac{Y(z)}{X(z)}\right|_{\text{zwp}} = \frac{1}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}} = 3 \frac{1}{1 - \frac{1}{2} z^{-1}} - 2 \frac{1}{1 - \frac{1}{3} z^{-1}}, \; |z| > \frac{1}{2}$$

$$h[n] = Z^{-1}\{H(z)\} = \left[3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n \right] u[n]$$

Obliczymy odpowiedź $y[n]$ systemu na pobudzenie $x[n]=10u[n]$ dwoma metodami.

A.Przekształcenie Z.

$$x[n]=10u[n] \quad \Leftrightarrow \quad X(z)=\frac{10}{1-z^{-1}} \qquad u[n]\stackrel{\Delta}{=}\begin{cases}1, & n\geq 0\\ 0, & n<0\end{cases}$$

$$H(z)=\frac{1}{1-\frac{1}{2}z^{-1}}\,\frac{1}{1-\frac{1}{3}z^{-1}}$$

$$X(z) \longrightarrow \boxed{H(z)} \longrightarrow Y(z)$$

$$Y(z)=X(z)H(z)=\frac{10}{\left(1-z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}=$$

$$=30\frac{1}{1-z^{-1}}-30\frac{1}{1-\frac{1}{2}z^{-1}}+10\frac{1}{1-\frac{1}{3}z^{-1}}$$

$$y[n]=\left[30-30\left(\frac{1}{2}\right)^{n}+10\left(\frac{1}{3}\right)^{n}\right]u[n]$$

B. Splot liniowy.

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = 10\sum_{k=-\infty}^{\infty}\left[3\left(\frac{1}{2}\right)^{k} - 2\left(\frac{1}{3}\right)^{k}\right]u[n-k] =$$

$$\underset{n\geq 0}{=} 10\sum_{k=0}^{n}\left[3\left(\frac{1}{2}\right)^{k} - 2\left(\frac{1}{3}\right)^{k}\right] = 10\left[\frac{3\left(1 - \left(\frac{1}{2}\right)^{n+1}\right)}{1-\frac{1}{2}} - \frac{2\left(1 - \left(\frac{1}{3}\right)^{n+1}\right)}{1-\frac{1}{3}}\right]$$

$$y[n] = \begin{cases} 10\left[3 - 3\left(\frac{1}{2}\right)^{n} + \left(\frac{1}{3}\right)^{n}\right], & n \geq 0 \\ 0, \quad n < 0 \end{cases}$$

Wykorzystano tu wzór na sumę malejącego postępu geometrycznego z wyrazami

$$a_1, a_2, \cdots, a_n$$

i ilorazem postępu

$$q = \frac{a_k}{a_{k-1}}$$

wynoszącą

$$S_n = \frac{a_1\left(1 - q^n\right)}{1 - q}, \quad \text{gdy } q \neq 0, 1 \text{ i } a_1 \neq 0$$